Spatial competition with intermediaries*

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Abstract

We consider a market with two horizontally differentiated products located at endpoints of the Hotelling line. We have previously shown that the optimal monopolist solution involves selling lotteries. We offer a model of competition allowing for emergence of third-party players, intermediaries, who may offer lotteries. Our general protocol asks (possibly many) intermediaries to approach the two firms with suggestions for lotteries, which include their composition, price for consumers, and revenue shares. Then, firms decide which lotteries to participate. Lotteries to which both firms participate are offered to the market, firms set their own prices, and demands realize. We consider multiple scenarios, including competitive and for-profit intermediaries, proportional and arbitrary sharing rules. We show that intermediaries emerge, which is typically beneficial for the firms. The consumer surplus may also increase with intermediaries. With proportional sharing rule a single lottery (1/2-1/2) is optimal for firms with competitive intermediaries. With arbitrary sharing rules, for some parameter values, the perfect cartel solution can be realized.

1 Introduction

In this paper we offer a new approach to model competition between firms producing horizontally differentiated goods. The product characteristic is horizontal if different customers have different ideal values of it. Location of hotels, product design, risks of financial securities, and food components are all examples of horizontal attributes.

Previously, we have solved the problem of the monopolist selling two horizontally differentiated products (Balestrieri, Izmalkov & Leao (2015)). We have shown that in the optimal mechanism the monopolist may want to offer various lotteries to consumers in addition to pure products. The monopolist price discriminates by offering lotteries at a relatively low prices to relatively indifferent consumers, which allows to charge higher prices to the customers who strongly prefer one of the products.

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The challenge to modelling duopolistic market consists in how to think about possible emergence and usage of lotteries, given that: (i) any two-good lottery requires involvement of both firms; (ii) the monopolist's solution offers a natural benchmark to compare duopoly to and for firms to aim for in any collusive agreement; (iii) lotteries of the optimal solution are priced differently (lower) and so cannot be realized from the pure-good offerings by the firms themselves; and (iv) opaque goods which can be thought of as lotteries are offered in practice (e.g., Hot Rate hotels deals by hotwire.com).

To model and analyze the competition between the two firms we allow for emergence of lotteries as separate products. To focus on competition and interaction with intermediaries we consider the simplest horizontally differentiated market, the Hotelling line.

In our general interaction protocol, one or many intermediaries propose lotteries or bundles of lotteries to firms. The firms then simultaneously decide which proposals to support. The lotteries supported by both firms are offered to the market. Finally, firms set their own prices, and consumers make their purchases. This protocol permits to analyze a variety of scenarios, allowing for arbitrary partition of surplus among the firms and intermediaries from lotteries offered to the market. In particular, we consider extreme market compositions in detail: competitive intermediaries leaving all extra surplus from lotteries to firms and the monopolist intermediary maximizing its profits subject to firms' participation. Crucially, we do not allow contingent deals or contracts on other prices, on actions of the other participants, on aggregate sales, etc., as with such options a perfect cartel solution can be easily achieved.

We show that lotteries emerge for intermediate base consumption values for goods. Remarkably, in the solutions optimal for firms or for the monopolist intermediary, a single lottery (1/2-1/2) is offered under most parameter values if any lottery is offered at all. For low values, the firms serve their own niche markets and the market is not fully covered. For high values, the firms' desire to undercut with own price is too strong and the pure price competition outcome obtains.

We also identify two specific elements of lottery designs that affect what firms and intermediaries can achieve. The first one is a profit sharing rule. We characterize solutions for proportional sharing rules, where a lottery price paid by customers is split among the firms proportionally to allocation probabilities, and for arbitrary sharing rules. The second one is whether the budget balance (or surplus) for any intermediary has to be satisfied on per lottery or per bundle of lotteries.

For some base consumption values, the perfect cartel solution can be implemented with a support of the sharing rule in which each firm gets 100% of the revenue from "nearby" lottery consumers, as this limits firms' incentives to undercut lottery demand with own good prices. Bundling of lotteries, where an intermediary may loose some revenue on some lotteries and gain on others, can increase overall producer surplus.

Overall welfare effects from emergence of lotteries are ambiguous. Lotteries increase welfare when market becomes fully covered, but also have a negative impact on those customers who would otherwise buy a pure product. Interestingly, the monopolist intermediary can shift some surplus from firms to consumers for some parameter values. And, separately, the monopolist intermediary can make firms worse off relative to the world without intermediaries by "threatening" with the worst out of multiple competitive equilibria if they do not agree to the lottery proposal.

2 Model

We consider a spatial competition model with two firms and a continuum of customers. The two firms are located at the extremes of a Hotelling line, each producing its own good. Costs of production are normalized to 0. Consumers are uniformly distributed along the line. Each consumer's location on the line is his private information. Firms and their goods are indexed by $i \in \{0, 1\}$, and consumers are indexed by $x \in [0, 1]$. Firms maximize profits.

Consumers demand at most one unit of the good, and the utility to consumer x from good i purchased at price p is

$$U(x, i, p) = V - c(|x - i|) - p,$$

where V is the base consumption value from either good, and $c(\cdot)$ is the cost function; we assume that the cost function is linear $c(y) = y$ for all $y \in [0, 1]$. Cost function can be interpreted as the cost of traveling consumer x has to incur to reach location i, or as the loss in utility due to consuming a good with a non-ideal configuration in some characteristics space.

A lottery $l = \langle (q_0, q_1); p; (s_0, s_1) \rangle$ is a triple consisting of an allocation rule, price, and revenue sharing rule: it supplies good 0 with probability q_0 or good 1 with disjoint probability q_1 at price p to consumers, out of which $s_0 p$ goes to firm 0 and $s_1 p$ to firm 1, respectively. We will only consider non-degenerate lotteries, with $q_0 \geq 0, q_1 \geq 0$, and $0 < q_0 + q_1 \leq 1$.

The expected utility for lottery $l = \langle (q_0, q_1); p; (s_0, s_1) \rangle$ is $U(x, l) = V - q_0 x - q_1(1 - x) - p$. A lottery is *sure-prize* if $q_0 + q_1 = 1$, is *one-good* if either $q_0 = 0$ or $q_1 = 0$, and is *two-good*, otherwise. A sharing rule is *proportional* if $s_0 = s\frac{q_0}{q_0+q_1}$ and $s_1 = s\frac{q_1}{q_0+q_1}$ for some $s \in [0, 1]$. It is *purely proportional* if it is proportional with $s = 1$.

In what follows, for notational convenience, when referring to lotteries we may omit some of the components of their triples, whenever the price and/or the sharing rule are either not important for arguments, or can be trivially inferred or derived. For example, we may write $\langle (q_0, q_1); p \rangle$ or $\langle q_0, q_1 \rangle$. For the allocation rule, we may also use generic (q_i, q_{-i}) for $i = 0, 1$.

We assume that intermediaries are necessary to set up and sell two-good lotteries and they can do that at no cost. We model the interaction between intermediaries and firms according to the following protocol.

At time $t = 1$, intermediaries approach both firms and offer to sell two-good lotteries. Each intermediary m offers a bundle of two-good lotteries $B_m = \{l^j\}_{j \in J_m}$, where j is an index from the index set J_m. Firms decide simultaneously whether to accept or reject the proposed bundles B_m. By accepting a bundle B_m, a firm commits to supply her good as prize in all lotteries l_j with $j \in J_m$, at no additional cost for intermediary m. For each m, if B_m is accepted by both firms, then all of its lotteries are offered to the market.

At time $t = 2$, after learning which two-good lotteries are sold in the market, each firm i decides on whether to offer own good at price p_i and/or one-good lotteries $l_i^j = \langle (q_i^j, 0); p_i^j; (1, 0) \rangle$.

Finally, at time $t = 3$, consumers make their purchasing decisions. We assume that each consumer can buy at most one lottery and no resale is possible.

For game-theoretic analysis we would impose some additional technical requirements to ensure the existence of each firm's best response given the offered lotteries (e.g. that each bundle offered is a closed set, and that any union of the bundles is a closed set). The equilibrium notion we use throughout the paper is subgame-perfect-equilibrium.

3 Preliminaries

3.1 No intermediaries

As benchmark, we derive the equilibrium in a duopoly environment where intermediaries are not present, and so without two-good lotteries. Firms compete by simultaneously setting the prices of their base goods and possibly offering one-good lotteries; consumers then decide which good or lottery (if any) to buy.

There are three regimes depending on the value of V: for low V, firms serve their niche markets as local monopolies, and there are some consumers that do not buy any goods; for high V, each firm serves half of the market, and equilibrium prices are independent of V; for intermediate V, there are multiple equilibria. In such intermediate region, prices are set so that the marginal consumer is indifferent between purchasing either good or not buying anything, but the location of the marginal consumer need not be $x = \frac{1}{2}$. That is, there exist asymmetric equilibria even though the model is symmetric.

Proposition 1 *The duopoly game has the following equilibria.*

1. *For $V \leq 1$, $p_0 = p_1 = \frac{V}{2}$. Each firm serves a niche market. Consumers $x \leq \frac{V}{2}$ buy good 0, consumers $x \geq 1 - \frac{V}{2}$ buy good 1.*[1]
2. *For $1 \leq V \leq \frac{3}{2}$, there are multiple equilibria. For $V \leq \frac{6}{5}$ any $x^* \in \left[1 - \frac{V}{2}, \frac{V}{2}\right]$ can be supported as the threshold customer. For $V \geq \frac{6}{5}$ any $x^* \in \left[\frac{V}{3}, 1 - \frac{V}{3}\right]$ can be supported. Consumers $x \leq x^*$ buy good 0 at price $p_0 = V - x^*$, consumers $x \geq x^*$ buy good 1 at price $p_1 = V - (1 - x^*)$.*
3. *For $V \geq \frac{3}{2}$, $p_0 = p_1 = 1$. Consumers $x \leq \frac{1}{2}$ buy good 0, consumers $x \geq \frac{1}{2}$ buy good 1.*

Proof. The proofs of all results can be found in Appendix ??. ■

[1] Throughout the paper we will use weak inequalities (or closed sets) to describe demands for products. Consumers belonging to more than one set are indifferent among (typically) two opportunities, and can select any of them.

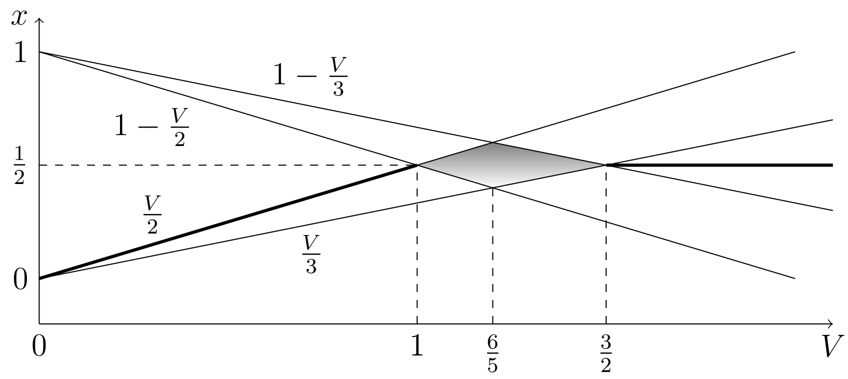

Figure 1: Unique equilibrium for $V < 1$, furthest type served by firm 0, $x^* = \frac{V}{2}$, is shown. Multiple equilibria for $1 \leq V \leq \frac{2}{3}$, possible range of x^* given V is shaded. Unique equilibrium $x^* = \frac{1}{2}$ for $V > \frac{3}{2}$.

Two points regarding the proposition we would like to highlight. First, there are multiple asymmetric equilibria for intermediate values of V, $V \in \left[1, \frac{3}{2}\right]$. This was previously shown by **?**. Since payoffs of these equilibria are continuation payoffs following the subgame in which one or both of the firms reject all lotteries, the worst asymmetric equilibrium payoff can be used to discipline firms to agree to lotteries. And, in particular, for this range of values, it would be possible to have equilibria in which firms becoming worse off in competition with intermediaries relative to some equilibrium payoffs (e.g. the symmetric equilibrium) without intermediaries.

Second, even though firms could offer one-good lotteries, in equilibrium they do not. When computing best replies, each firm acts as a residual monopolist facing consumers with a weakly-increasing outside option in distance from its location. By a simple extension of Myerson (1981) or Riley & Zeckhauser (1983) results, the best replies have no lotteries. As we shall see, with lotteries on the market, it may happen that firms would face consumers with decreasing outside option, in which case one-good lotteries may become part of firms' best responses.

Given the equilibria, we underline few results that we will use as terms of comparison in the next sections.

For later comparisons with other competition regimes, we collect some of the numerical results in one place. We use superscript *ni* to denote no intermediaries case.

Corollary 1 *The market is fully covered, i.e. all consumers buy a good, when* $V \geq V^{ni} = 1$. *For* $V < V^{ni}$*, the market is not fully covered.*

Equilibrium producer, consumer, and total surpluses are as follows.

1. *For* $V \leq 1$, $\pi_0^{ni} = \pi_1^{ni} = CS^{ni} = \frac{1}{4}V^2$, $TS^{ni} = \frac{3}{4}V^2$.

2. *For* $1 < V \leq \frac{6}{5}$, $\pi_0^{ni}, \pi_1^{ni} \in \left[\left(1 - \frac{1}{2}V\right)\left(\frac{3}{2}V - 1\right), \frac{1}{4}V^2\right]$, $CS^{ni} \in \left[\frac{1}{4}V^2 - \frac{1}{2}V + \frac{1}{2}, \frac{1}{4}\right]$*, and*

$TS^{ni} \in \left[\frac{1}{2}V(V-1) + \left(1-\frac{1}{2}V\right)\left(\frac{3}{2}V-1\right) + \frac{1}{2}, V-\frac{1}{4}\right]$.

3. *For* $\frac{6}{5} < V < \frac{3}{2}$, $\pi_0^{ni}, \pi_1^{ni} \in \left[\frac{2}{9}V^2, \left(1-\frac{1}{3}V\right)\left(\frac{4}{3}V-1\right)\right]$, $CS^{ni} \in \left[\frac{1}{9}V^2 - \frac{1}{3}V + \frac{1}{2}, \frac{1}{4}\right]$, *and* $TS^{ni} \in \left[\frac{1}{3}V^2 - \frac{1}{3}V + \left(1-\frac{1}{3}V\right)\left(\frac{4}{3}V-1\right) + \frac{1}{2}, V-\frac{1}{4}\right]$.
   In the symmetric equilibrium for $1 < V < \frac{3}{2}$, $\pi_0^{ni} = \pi_1^{ni} = \frac{1}{2}\left(V-\frac{1}{2}\right)$ *for* $i = 0, 1$, $CS^{ni} = \frac{1}{4}$, $TS^{ni} = V - \frac{1}{4}$.

4. *For* $V \geq \frac{3}{2}$, $\pi_0^{ni} = \pi_1^{ni} = \frac{1}{2}$, $CS^{ni} = V - \frac{5}{4}$, $TS^{ni} = V - \frac{1}{4}$.

Notably, the producer, consumer, and total surpluses in any asymmetric equilibrium is lower than in the symmetric equilibrium.

3.2 Monopolist's solution

The solution is derived in Balestrieri et al. (2015).

Proposition 2 *The optimal selling mechanism under perfect cartel is the following:*

$$\mu^{pc} = \begin{cases} \mu^{bg}, & \text{for } V < \frac{1}{2}, \\ \mu^{l}, & \text{for } V > \frac{1}{2}. \end{cases}$$

where

$$\mu^{bg} = \begin{cases} \left\langle (1,0); \frac{V}{2} \right\rangle, & \text{for } x < \frac{V}{2}, \\ \left\langle (0,1); \frac{V}{2} \right\rangle, & \text{for } x > 1 - \frac{V}{2}, \end{cases} \qquad \mu^{l}(y) = \begin{cases} \left\langle (1,0); V - \frac{1}{4} \right\rangle, & \text{for } x < \frac{1}{4}, \\ \left\langle \left(\frac{1}{2}, \frac{1}{2}\right); V - \frac{1}{2} \right\rangle, & \text{for } \frac{1}{4} < x < \frac{3}{4}, \\ \left\langle (0,1); V - \frac{1}{4} \right\rangle, & \text{for } x > \frac{3}{4}. \end{cases}$$

Thus, the perfect cartel uses $\left\langle \frac{1}{2}, \frac{1}{2} \right\rangle$ lottery once the base-consumption value is sufficiently high. Only one lottery is sufficient to maximize the expected revenues. When the lottery is offered, the market is fully covered. Comparing the perfect cartel solution and the no intermediary equilibria, it is immediate to observe that, under perfect cartel, the market becomes fully covered at value $V^{pc} = \frac{1}{2}$, a lower threshold than the one for competition without intermediaries, $V^{ni} = 1$. No surplus is left to consumers who buy the lottery. Instead, in equilibrium, consumers who buy base goods earn informational rents.

Again, for later comparisons we state:

Corollary 2 *The producer, consumer, and total surpluses in the optimal perfect cartel mechanism are as follows.*

1. *For* $V < \frac{1}{2}$, $PS^{pc} = \frac{1}{2}V^2$, $CS^{pc} = \frac{1}{4}V^2$, *and* $TS^{pc} = \frac{3}{4}V^2$.

2. *For* $V \geq \frac{1}{2}$, $PS^{pc} = V - \frac{3}{8}$, $CS^{pc} = \frac{1}{16}$, *and* $TS^{pc} = V - \frac{5}{16}$.

While offering the lottery brings more profits to the firms in the perfect cartel solution, the welfare implications are ambiguous. There are two opposite effects from using lotteries: (i) an increase in market coverage, which is positive; and (ii) allocating not the most preferred good to some consumers, which is negative. Therefore, the total surplus is higher under perfect cartel for $\frac{1}{2} < V < \frac{5}{6}$ relative to the equilibrium total surplus in competition without intermediaries, and it is lower for $V > \frac{5}{6}$.

3.3 Perfect cartel implementation with intermediaries

There are several possibilities how firms can implement the perfect cartel with intermediaries. To be completed.

Possible schemes: contract on prices/ quantities directly (the price of the lottery tied to the price of the pure good(s)); there is a compensation to one of the parties depending whether a certain amount of goods sold, or lotteries (enough if at least one lottery is bought by consumers), etc..

4 Competition with intermediaries

4.1 Simple lotteries and examples

First, we consider lottery $\left\langle \frac{1}{2}, \frac{1}{2} \right\rangle$ that is the only two-good lottery featured in the selling mechanism that maximizes the perfect cartel's profit. We also suppose that there is a single intermediary offering the lottery.

Proposition 3 *The equilibria of the duopoly game in which an intermediary offers lottery $\left\langle \left(\frac{1}{2}, \frac{1}{2}\right); p_l; \left(\frac{1}{2}, \frac{1}{2}\right) \right\rangle$ are described below, specifying supportable lottery price p_l, firms' prices p_0, p_1 and allocations in the case the lottery is demanded in equilibrium. In the subgame following rejection of the lottery by one of both firms, one of the equilibria described in Proposition 1 is played.*

1. *For $\frac{1}{2} \leq V \leq 1$, $p_l \in \left[\underline{p}, V - \frac{1}{2}\right]$, and $p_0 = p_1 = \frac{3}{4}p_l + \frac{1}{4}$; consumers $x \leq x^*$ buy good 0, consumers $x \geq 1 - x^*$ buy good 1, and consumers $x \in [x^*, 1 - x^*]$ buy the lottery, where $\underline{p} = 4 - \sqrt{17 - 4V^2}$ and $x^* = \frac{1}{4}p_l + \frac{1}{4}$.*

2. *For $1 < V \leq \frac{3}{2}$, $p_l \in \left[\underline{p}, V - \frac{1}{2}\right]$, and $p_0 = p_1 = \frac{3}{4}p_l + \frac{1}{4}$; consumers $x \leq x^*$ buy good 0, consumers $x \geq 1 - x^*$ buy good 1, and consumers $x \in [x^*, 1 - x^*]$ buy the lottery, where $\underline{p} = 4 - \sqrt{17 - 16\left\{\max\left\{\pi_0^{ni}, \pi_1^{ni}\right\}\right\}}$, $x^* = \frac{1}{4}p_l + \frac{1}{4}$, and the firms play the asymmetric equilibrium with profits π_0^{ni} and π_1^{ni} in the subgame following rejection of the lottery.*

3. *for all V, there are equilibria equivalent to those in Proposition 1, in which both firms reject the lottery, or the lottery is accepted but is not demanded, $p_l \geq \max\{0, p_0, p_1\}$.*

In the next example we show that non-sure prize lotteries can be supported as an equilibrium outcome

Example 1 *Consider* $V = \frac{3}{4}$*, and an intermediary offering lottery* $\left\langle \left(\frac{2}{5}, \frac{2}{5}\right)\right); \frac{1}{5}; \left(\frac{1}{2}, \frac{1}{2}\right)\rangle$*. Firms accept the lottery at* $t = 1$ *and sell their base goods at price* $p_0 = p_1 = \frac{17}{40}$ *at* $t = 2$*. Firms' profits are* $\pi_0 = \pi_1 = 0.17 > \pi^{ni} = 0.14$*. Consumers* $x \in \left[0, \frac{13}{40}\right]$ *buy good* 0*, consumers* $x \in \left[\frac{27}{40}, 1\right]$ *buy good* 1*, consumers* $x \in \left[\frac{13}{40}, \frac{27}{40}\right]$ *buy the lottery.*

The following example shows that an asymmetric lottery can be supported and that the market may not be fully covered at the same time.

Example 2 *Consider* $V = \frac{3}{4}$*, and an intermediary offering lottery* $\left\langle \left(\frac{2}{3}, \frac{1}{3}\right); \frac{1}{4}; \left(\frac{1}{2}, \frac{1}{2}\right)\right\rangle$*. Firms accept the lottery at* $t = 1$ *and sell their base goods at price* $p_0 = p_1 = \frac{3}{8}$*. The profits are* $\pi_0 = 0.148$*, and* $\pi_1 = 0.203$*. Consumers* $x \in \left[0, \frac{5}{16}\right]$ *buy good* 0*, consumers* $x \in \left[\frac{5}{8}, 1\right]$ *buy good* 1*, consumers* $x \in \left[\frac{5}{16}, \frac{1}{2}\right]$ *buy the lottery, consumers* $x \in \left[\frac{1}{2}, \frac{5}{8}\right]$ *do not buy any good.*

The last example shows that one-good lotteries can be optimally offered by firms in equilibrium. This is driven by the decreasing outside option for consumers from the perspective of firm 0 generated endogeneously by the bundle of lotteries.

Example 3 *Consider* $V = 0.55$ *and an intermediary offering a bundle of lotteries* $B = \left\{\left\langle \left(\frac{13}{20}, \frac{7}{20}\right); 0.116\right\rangle, \left\langle \left(\frac{1}{4}, \frac{3}{4}\right); 0.05\right\rangle\right\}$ *with proportional sharing rule and all the proceeds going to firms. Firms accept the bundle at* $t = 1$*, and sell their goods at* $p_0 = 0.273\,49$ *and* $p_1 = 0.168\,75$*. In addition, Firm* 0 *sells one-good lottery* $\left\langle \left(\frac{3}{10}, 0\right); 0.081\right\rangle$*. The profits are* $\pi_0 = 0.0786$*, and* $\pi_1 = 0.0532$*. Consumers* $x \in [0, 0.275]$ *buy good* 0*, consumers* $x \in [0.275, 0.28]$ *buy the one-good lottery, consumers* $x \in [0.5, 0.737\,5]$ *buy lottery* $\left\langle \frac{1}{4}, \frac{3}{4}\right\rangle$*, consumers* $x \in [0.737\,5, 1]$ *buy good* 1*, and consumers* $x \in [0.28, 0.5]$ *do not buy any good.*

4.2 Competitive intermediaries

Description/ discussion of what are competitive intermediaries. (1) firms "create" an intermediary under break-even condition (per lottery) and we solve for maximal revenue that the firms may obtain; (2) there is a continuum of intermediaries offering each lottery (and this necessesitates budget balance per lottery); (3) there is a contunuum of intermediaries offering a bundle (and so competition is on per bundle level); still this would generate budget balance per lottery [as budget check happens only in equilibrium]

4.2.1 Proportional sharing rule

Proposition Only lottery $\left\langle \left(\frac{1}{2}, \frac{1}{2}\right); V - \frac{1}{2}; \left(\frac{1}{2}, \frac{1}{2}\right)\right\rangle$ is offered in the optimal from the perspective of the firms solution **with proportional sharing rule** and only for $V \in \left[\frac{1}{2}, \frac{3}{2}\right]$.

Intuition: Key element is that an introduction of any asymmetric lottery due to an effect on consumers creates even stronger incentive for any firm to undercut with own good.

4.2.2 Arbitrary sharing rule

Crucial Lemma: (exact formulation to be decided) Suppose lottery $l = \langle (q_0, q_1); p; (s_0, s_1) \rangle$ is demanded by consumers in set $X \subset [0,1]$, then it can be arbitrarily well approximated by the bundle of lotteries $l(x) = \langle (q_0, q_1); p; (s_0(x), s_1(x)) \rangle$ for $x \in X$, with the same probabilities and price, but arbitrary sharing rule depending on the consumer type. Approximated in the sense each type x would purchase $l(x)$ instead of l.

The implication of the Lemma is that we can use consumer-type specific lotteries, in particular setting revenue sharing rule (which consumers do not care about) arbitrarily.

Proposition 4 *With competitive intermediaries and arbitrary sharing rule, the firms can implement the perfect cartel solution for* $V \in \left[\frac{1}{2}, \frac{3}{2}\right]$, *with lotteries* $l^0(x) = \langle \left(\frac{1}{2}, \frac{1}{2}\right); V - \frac{1}{2}; (1,0) \rangle$ *for* $x \in \left[\frac{1}{4}, \frac{1}{2}\right]$ *and* $l^1(x) = \langle \left(\frac{1}{2}, \frac{1}{2}\right); V - \frac{1}{2}; (0,1) \rangle$ *for* $x \in \left[\frac{1}{3}, \frac{3}{4}\right]$.

Proposition 5 *When* $V > \frac{3}{2}$, *if intermediaries offer bundle* B_n, *the equilibrium of the duopoly game is such that firms accept the bundle and price their goods* $p_0^{B_n} = p_1^{B_n} = p_l - \frac{1}{2}q + \frac{1}{2}$. *Consumers* $x \in \left[0, \frac{1}{4}\right]$ *buy good* 0, *consumers* $x \in \left[\frac{1}{4}, \frac{1}{2}\right]$ *buy lottery* $\langle q, 1-q \rangle$, *consumers* $x \in \left[\frac{1}{2}, \frac{3}{4}\right]$ *buy lottery* $\langle 1-q, q \rangle$, *and consumers* $x \in \left[\frac{3}{4}, 1\right]$ *buy good* 1.

5 Monopolist (Exploitative) Intermediary

1. Proportional sharing rule with lottery $\langle \frac{1}{2}, \frac{1}{2} \rangle$. (as an example; somewhere was computed)

2. Proportional sharing rule/ any lottery/ bundle. The monopolist can benefit from using bundles.

3. Arbitrary sharing rule (per lottery budget balance): Perfect cartel implementation for $V < 1$. Firms become worse-off for $V \in \left[1, \frac{3}{2}\right]$ (due to multiple equilibria). For $V > \frac{3}{2}$ – open question.

6 Per bundle budgets

(see different file)

In the competitive intermediaries case firms can improve by easing budget constraints for lotteries on the per bundle levels.

7 Final Remarks

endogenous quality/ investment

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